PART I

INTRODUCTION

CHAPTERS

1. SETTING AND SUMMARY OF DISSERTATION
2. SETTING AND SIGNIFICANCE OF THE CAUSAL STATISTICS PROJECT
3. IMPORTANCE OF THE DISSERTATION

Preliminary Remarks

Part I is a presentation of introductory material. It is designed to give the reader an understanding of the origin, setting, and importance of the dissertation. Also, a chapter-by-chapter summary of the dissertation is included (Subsection 1.4).

CHAPTER 1

SETTING AND SUMMARY OF DISSERTATION

Sections

1.1 Setting of the Dissertation

This dissertation is the initial portion of a much larger research project, referred to as the causal statistics project. The full title of the causal statistics project is Causal Statistics: The Derivation, Operationalization, Computerization, and Application.

Causal statistics is a mathematical inquiring system which enables empirical researchers to draw causal inference from non-experimental, quasi-experimental, and imperfectly experimental data (almost all social science and much medical research data fall into one of these three categories); based upon the least restrictive assumptions possible. Econometrics could be considered to be a rudimentary form of causal statistics; but the research proposed in the causal statistics project will go far beyond the present state of knowledge in econometrics. Also, the causal interpretation of econometrics is not well explored.

The dissertation--being the fundamental stage of the causal statistics project--deals with the philosophical background of the concept of causality and then derives--from a set of basic axioms about the operation of the universe--a general formulation of causal statistics. This formulation can be considered to be a generalized template which fits any form of causal relationship among any number of variables.

1.2 Origin of the Causal Statistics Project

The causal statistics project originated from an attempt to make causal inferences in an empirical, non-experimental management study. We were attempting to study the effect of ecological variables (not to be confused with those variables of interest to the Sierra Club or Life of the Land) on the productivity of scientific research and development.

Since the study was non-experimental, causation was difficult to establish. We attempted to use the causal inquiring systems available at that time, but could not apply them with understanding, insight or confidence--e.g., the assumptions implicit in the various systems were unknown.

We then turned our efforts to the development of a causal inquiring system which could overcome the aforementioned problems. As this work proceeded, we saw the far-reaching importance of this line of statistical research. Its significance dwarfed that of the original R & D study. For this reason the R & D study was discontinued, with the causal statistics project taking its place.

Originally the dissertation was intended to encompass the whole causal statistics project. But, after writing began, it was obvious that the project was too expansive for a single dissertation. Hence it was decided that only the initial portion of the causal statistics project would be included in the dissertation.

1.3 Outline of the Causal Statistics Project

The goal of the causal statistics project is to perform the research necessary to write a book, entitled Causal Statistics: The Derivation, Operationalization, Computerization, and Application. A preliminary table of contents for such a book is presented in Table 1-1.

The proposed book contains eight parts. The first three parts are essentially the same as the first three parts of the dissertation. Part I is the introduction. Part II discusses the philosophical background of causality. Part III sets forth some basic axioms about the operation of the universe and--from this axiom system and later assumptions--derives a very general formulation of causal statistics. Part IV of the dissertation is entitled Concluding Remarks. Of course, in the book the concluding remarks are presented in Part VIII, the last part.

Table 1-1

CAUSAL STATISTICS:

THE DERIVATION, OPERATIONALIZATION,

COMPUTERIZATION AND APPLICATION

TABLE OF CONTENTS

Part IV of the book presents some of the simple aspects of the application of causal statistics. This part is intended to give the reader an understanding of the process of the application of causal statistics. Part V develops a number of mathematical formulations designed to facilitate the application and interpretation of causal statistics. In Part VI the computerization of causal statistics is presented. In Part VII causal statistics is applied, in a number of different ways, to an empirical study. Then, the various sets of results are compared. Part VIII completes the book with some concluding remarks.

1.4 Summary of Dissertation

This section is a chapter by chapter summary of the dissertation. To avoid excessive length, the content of this summary is limited in scope and glosses over or omits many important points. Sometimes precise accuracy is sacrificed in deference to clarity and/or brevity.

The dissertation is composed of four parts and 14 chapters, as indicated by the table of contents. Following the table of contents is a reference list of symbols and definitions. The first three chapters are introductory.

Chapter 1 discusses the setting of the dissertation and summarizes it, chapter by chapter. The dissertation is the initial portion of a causal statistics research project, entitled Causal Statistics: The Derivation, Operationalization, Computerization, and Application. Causal statistics is a mathematical inquiring system which enables empirical researchers to draw causal inference from non-experimental, quasi-experimental, and imperfectly experimental data, based upon the least restrictive assumptions possible. This causal statistics project originated from an attempt to make causal inferences from non-experimental data in an R & D management study. An outline of the causal statistics project is presented in the form of a table of contents in Table 1-1.

The setting and significance of the causal statistics project are considered in Chapter 2. A diagramatic representation of the research process is presented. Then it is shown how causal statistics, causal theories, and causal predictions fit into the research process. Section 2.4 shows that causal statistics is an extremely important analytical tool for non-experimental, empirical research--e.g., social and medical sciences research. Existing statistical techniques for causal inquiries are discussed in Section 2.5 and compared in the next. The final section investigates the need for further research in the field of causal statistics. It concludes that further work is needed in the areas of foundations, operationalization, computerization, and communication. Operationalization is an attempt toward facilitating the application and/or enhancing the applicability of the analytical tool by way of mathematical formalization.

Chapter 3 is an argument for the importance of the dissertation. Its importance can be viewed from two different perspectives. First, it can be argued that since the causal statistics project is important--as is concluded in Chapter 2--and since the research performed in the dissertation is a necessary part of the causal statistics project, then the dissertation research is a valuable contribution. Second, one could consider the dissertation's own inherent significance, irrespective of its relationship with the causal statistics project. This chapter develops both approaches.

The philosophical background of the concept of causality is discussed in Part II. Chapter 4 presents and analyzes the definitions of cause forwarded by Galileo, Hume, and Mill. Their definitions are operational rather than theoretical (i.e., ontological) and encompass all combinations of necessary and/or sufficient conditions. Each of these definitions is shown to be inconsistent with the common usage of the term cause. We then forward two theoretical definitions of cause (called cause(P)) which are consistent with the common usages of the term.

> Definition (4-1): X causes(P) Y, if and only if the behavior of X produces a force on Y, where X and Y are specific events or behaviors.

> Definition (4-2): X causes(P) Y, if and only if the behavior of X produces a force on Y, where X and Y are classes of events.

Definition (4-1) is an event definition of cause and Definition (4-2) is a "law" definition of cause.

Chapter 5 discusses the causal philosophy of David Hume. Hume's two arguments are presented. They conclude that (1) it is impossible to be <u>certain</u> (100%) that any two objects are related causally and (2) even if one could determine that two objects were related causally in the past, it would be impossible to be certain that these two objects would continue to be related causally in the present or future. While Hume's two arguments are presented for his definition of cause, they are also valid for our theoretical, "law" definition of cause. But, contrary to the impressions of many philosophies and scientists, Hume's conclusions do not destroy the usefulness of the concept of causality and Hume admits that. There is little, if anything, that we can be philosophically certain about, including associations (i.e., correlations). But even a child will learn to avoid putting his hand into a flame because of his <u>uncertain</u> causal theory (based on experience) that a flame can cause a painful burn. It is a useful theory for the child and, in the same way, causal theories are useful for the scientist, the planner, and the practitioner. Causality is the only known and reasonable concept which can explain (i.e., is consistent with) observed phenomena in our universe.

Mill's five Canons of Induction or Methods are presented and analyzed in Chapter 6. These Canons are techniques for establishing causal connections. Mill claimed these Methods could <u>prove</u> causal connections. Although this statement was a little over exuberant--in light of Hume's arguments and some inherent flaws in the Methods--the Methods are important from a pragmatic point of view and do infer useful causal connections. The Methods are equally valid (or invalid) for both Mill's and our definitions of cause.

The major objective of Part III is to derive the most general formulation of causal statistics. Consider the typical econometric model, a system of simultaneous linear equations relating n variables. This is certainly not the most general way to express the causal connections among n discrete variables. What is the most general formulation? What axioms and assumptions is it based on? Are causal equations subject to the usually mathematical manipulations? These are the most important questions to be answered in Part III.

The most general formulation describing the causal relationships among n variables is called the <u>universal model of causal statistics</u>. The universal model is the beginning point for the application of causal statistics. In the application of causal statistics variables are placed into the universal model and it is specialized via simplifying assumptions, which are based on theory and/or intuition. Data are then inserted into the system and the parameters estimated, if the system is identified. This yields a causal theory relating the n variables, if the basic assumptions are reasonably well satisfied.

A general causal theory of the operation of the universe is forwarded in Chapter 7. Then, the theory is analyzed, axioms formalizing this theory are forwarded, and a mathematical description of this causal theory is formulated.

It is theorized that the universe is composed of fundamental objects or variables which approach a size of zero as a limit. The natural causal laws of the universe act between adjacent (i.e., touching)--in time and space --fundamental variables. These causal interactions are called <u>microcausal</u> interactions. Causal interactions between spatially and/or temporally separated objects are called <u>macrocausal</u> interactions. If a fundamental variable ϕ_1, is the macrocause of ϕ_2, then ϕ_1 and ϕ_2 must be connected by a chain of intervening, adjacent fundamental variables (Λ_i's), through which the causal impulse is transmitted. See Figure 7-2. This theory is

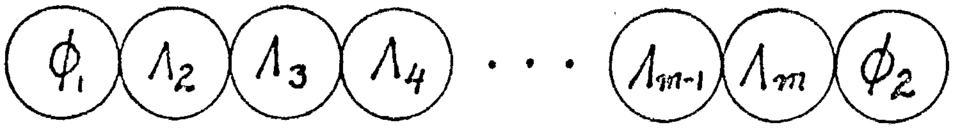

Figure 7-2

perfectly consistent with all observed phenomena.

Sections 7.5.1 and 7.5.2 each present an axiomatic foundation of this theory. Section 7.5.1 presents an intuitive axiomization while Section 7.5.2 presents a mathematical axiomization. The two axiom systems are identical in substance but different in form. The mathematical derivation proceeds from the mathematical axiom system, but, to more clearly convey our basic cosmology, we will state the intuitive axiom system.

Axiom (7-1): The operation of the universe is based on natural laws.

Axiom (7-2): The natural laws of the universe are microcausal laws.

Axiom (7-3): The natural laws of the universe are concerned only with relationships between adjacent and/or copositional fundamental variables.

Axiom (7-4): The natural laws of the universe are non-stochastic.

Axiom (7-5): There is no action (causation) at a distance in time and/or space.

Axiom (7-6): All microcausal impulses move at finite speeds.

Axiom (7-7): The natural laws of the universe are not functions of time and/or space and do not change over time and/or space.

Axiom (7-8): The natural laws of the universe operate identically between all fundamental objects. In other words, the natural properties of all fundamental objects are identical (see Subsection 7.2.3).

After forwarding the axioms, we formulate the mathematical description of the effect of a fundamental variable on one adjacent to it. See Figure 7-4. $\overline{\Delta l_1}$ is the distance vector between the centers of the two objects and Δt_1 is the time it takes the causal impulse to travel from $\overline{l_1}$ to $\overline{l_1} + \overline{\Delta l_1}$.

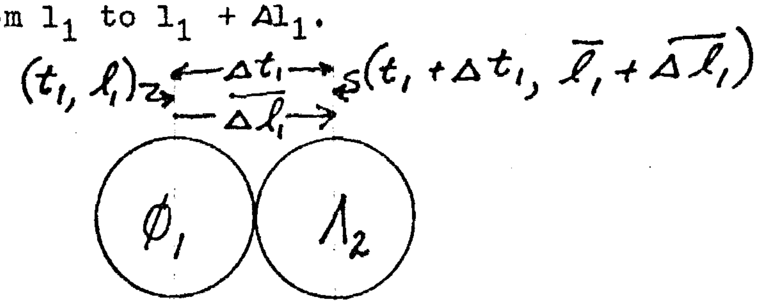

Figure 7-4

$$V_1^+\left[\Lambda_2(t_1 + \Delta t_1, \overline{l_1} + \overline{\Delta l_1})\right] \xleftarrow{mc} V_2^+\left[\emptyset_1(t_1, \overline{l_1}), \Lambda_2(t_1, \overline{l_1} + \overline{\Delta l_1})\right] \qquad (7\text{-}1)$$

" $\xleftarrow{mc}$ " means that the right side of the equation--called the causing side--is the microcause of the left side--called the caused side. V_1^+ and V_2^+ are two different universal mathematical operators. V_1^+, for example, can indicate any functional form and/or any differential or integral order of $\Lambda_2(t_1 + \Delta t_1, \overline{l_1} + \overline{\Delta l_1})$. This is a very simple formulation of microcausal micromathematics. The "micro" in micromathematics refers to the fact that the causing and caused variables are fundamental variables.

Chapter 7 closes by describing the macrocausal effect of $\emptyset_1$ on $\emptyset_2$ (see Figure 7-2) by writing a system of microcausal equations (like equation (7-1)) for the effect of $\emptyset_1$ on Λ_2, Λ_2 on Λ_3, Λ_3 on Λ_4, ..., Λ_{m-1} on Λ_m, and Λ_m on $\emptyset_2$. This is called contiguous causal micromathematics.

Chapter 8 derives discrete causal micromathematics from contiguous causal micromathematics. We assume away the need to consider the intervening fundamental variables and determine $\emptyset_2$ from only $\emptyset_1$. In doing this we make the following assumptions:

> Assumption (8-2): The configuration among the fundamental objects, on or adjacent to the connecting causal chains, is unchanged from run to run (i.e., it is the same on each run).
>
> Assumption (8-4): The immediately prior value of each relevant variable, along or adjacent to each causal chain, is the same on each run.
>
> Assumption (8-10): Any part of the behavior--of a variable adjacent to a causal chain--which is not ultimately caused by the value--after observation begins--of an exogenous variable (like $\emptyset_1$), is the same on each run.

These assumptions are very restrictive, but necessary if we wish to describe the behavior of the universe as a discrete process.

Chapter 9 generalizes discrete causal micromathematics to an m' variable system where each fundamental variable in the system has a causal effect on every other variable in the system. This generalization is called the <u>universal model of discrete causal micromathematics</u>.

The universal model of discrete causal micromathematics derived in Chapter 9 lacks direct relevance to social science or medical research because this model describes the causal relationships among fundamental variables and fundamental variables are, at present, unobservable. We categorize and think in terms of aggregates of fundamental variables, called <u>macrovariables</u>.

In Chapter 10 fundamental variables are aggregated to form macrovariables and causal chains are aggregated to form causal macrochains. Applying the universal model of discrete causal micromathematics to the component fundamental variables (Υ's) of two macrovariables, Z_1 and Z_2; we describe Z_2 in terms of Z_1. This result is discrete causal macromathematics and is attainable only after applying one or both of the following fairly restrictive assumptions:

> Assumption (10-4): The macrovariables, Z_i's, are defined such that the fundamental variables, Υ_{ij}'s, for all j, in the equations relating macrovariables are completely factorable in terms of (i.e., replaceable by) the Z_i's.
>
> Assumption (10-3): During each run the values of the fundamental variables composing Z_i have a specific (i.e., unchanging, constant) mathematical relationship to each other.

The "macro" in macromathematics refers to the fact that the formulation describes the relationships among macrovariables rather than fundamental variables.

The purpose of Chapter 11 is analogous to the purpose of Chapter 9--where a mathematical formulation involving one exogenous and one endogenous variables was generalized to form an m' variable, reciprocal causal model. The difference is that Chapter 11 generates a formulation representing causal connections among macrovariables via causal macrochains rather than causal connections among fundamental variables via causal chains. The result describes all possible macrocausal connections among n macrovariables and is called the

<u>universal model of discrete causal macromathematics</u>.

The universal model of causal statistics is similar to the universal model of discrete causal macromathematics. The difference is that the former allows for error in its formulation; the latter does not. Chapter 12 discussed the sources of error and their effects on error terms. Error vectors are then inserted into the universal model of discrete causal macromathematics to yield the <u>universal model of causal statistics</u>.

$$U_1\left[Z_1(t)\right] \longleftarrow U_1^*\left[t-t_{01},\ \overline{Z_1(t-\tau_{11})},\ \overline{Z_2(t-\tau_{21})},\ \ldots,\ \overline{Z_n(t-\tau_{n1})},\ Z_1(t_{01}),\ \bar{\epsilon}_1\right];$$

$$U_2\left[Z_2(t)\right] \longleftarrow U_2^*\left[t-t_{02},\ \overline{Z_1(t-\tau_{12})},\ \overline{Z_2(t-\tau_{22})},\ \ldots,\ \overline{Z_n(t-\tau_{n2})},\ Z_2(t_{02}),\ \bar{\epsilon}_2\right];$$

$$\vdots$$

$$U_n\left[Z_n(t)\right] \longleftarrow U_n^*\left[t-t_{0n},\ \overline{Z_1(t-\tau_{1n})},\ \overline{Z_2(t-\tau_{2n})},\ \ldots,\ \overline{Z_n(t-\tau_{nn})},\ Z_n(t_{0n}),\ \bar{\epsilon}_n\right]. \qquad (12\text{-}4)$$

The U's are universal mathematical operators which were discussed in our summary of Chapter 7. The Z's are macrovariables. The "$\longleftarrow$" means that the right-hand side is the cause of the left-hand side. t is time. The $t-t_{01}$ terms are transient terms, significant only in the initial stages of the application. τ_{ij} is the time lag with which Z_i effects of Z_j. $\overline{Z_i(t-\tau_{ij})}$ is a vector of Z_i's measured at various time lags. $Z_i(t_{0i})$ is the value of Z_i exactly dt before the first causal impulse--emitted from one of the Z's at the beginning of observation--reaches Z_1. The $\bar{\epsilon}_1$'s are the error vectors.

In Part IV, Concluding Remarks, Chapter 13 indicates that the dissertation has accomplished its purpose. It has given causal statistics a philosophical and mathematical underpinning. Also, Chapter 13 points out that, in addition, the dissertation makes potential contributions to the fields of philosophy, physics, and mathematics.

Chapter 14 makes some recommendations for future work. It first discusses further work needed in the field of the dissertation. Then further research needed in the field of causal statistics is discussed.