CHAPTER 9

DERIVATION OF THE UNIVERSAL MODEL OF DISCRETE CAUSAL MICROMATHEMATICS

Sections

9.1 Introduction

In the last chapter (Section 8.3) we ruled out consideration of exogenous variables other than $\emptyset_1$, as shown in Figure 8-1. In this chapter we explicitly investigate the implications of additional exogenous variables for our description of the universe via discrete causal micromathematics. The result is a mathematical description of the causal interconnections among m discrete, fundamental variables, called the universal model of discrete causal micromathematics.

9.2 Causal Chains Converging at $\emptyset_2$

In this section we will investigate the situation shown in Figure 9-1. The bottom portion of the figure represents Figure 8-2. It is a causal chain between $\emptyset_1$ and $\emptyset_2$. The hump on the chain indicates that the chain may contain causal loops like the ones in Figures 8-2, 8-3, and/or 8-4.

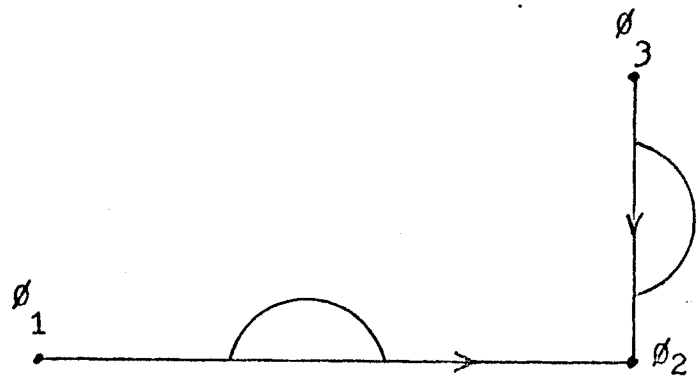

Figure 9-1

The right side of Figure 9-1 symbolizes a causal chain connecting $\emptyset_3$ and $\emptyset_2$. It also may have causal loops. To say that the causal chain contains causal loops is to say that there is more than one causal path connecting $\emptyset_3$ and $\emptyset_2$.

$\emptyset_1$ and $\emptyset_3$ are exogenous (i.e., are not macrocaused by any variable in the assumed relevant system), fundamental variables and $\emptyset_2$ is an endogenous (i.e., macrocaused by one of more variables in the assumed relevant system), fundamental variable.

Mathematically, the discrete causal connection between $\emptyset_1$ and $\emptyset_2$ is given by equation (8-11), which is

$$U\left[\emptyset_2(t)\right] \longleftarrow U\left[t-t'_{012},\ \overline{\emptyset_1(t-\theta_{12})},\ \emptyset_2(t'_{012})\right], \quad (9-1)$$

where V^*_∞ and V'''_∞ are each replaced by U.

U is used to represent any universal mathematical operator. The U on the left side of equation (9-1) does not represent the same mathematical functions and/or operations as does the U on the right side of the equation. U just means that some universal mathematical operator exists in that position. This U notation will be used in Chapters 9-12. Any time one U is necessarily the same as another, this fact will be specifically noted.

In an analogous manner to the derivation of equation (9-1), the macrocausal connection between $\emptyset_3$ and $\emptyset_2$ is described by

$$U\left[\emptyset_2(t)\right] \longleftarrow U\left[t-t'_{012},\ \overline{\emptyset_3(t-\theta_{32})},\ \emptyset_2(t'_{012})\right]. \quad (9\text{-}2)$$

Combining the effects of $\emptyset_1$ and $\emptyset_3$ on $\emptyset_2$, we get

$$U\left[\emptyset_2(t)\right] \longleftarrow U\left[t-t'_{02},\ \overline{\emptyset_1(t-\theta_{12})},\ \overline{\emptyset_3(t-\theta_{32})},\ \emptyset_2(t'_{02})\right], \quad (9\text{-}3)$$

where $\underline{t'_{02}}$ is the instant before the first causal impulse from $\emptyset_1(t_0)$ or $\emptyset_3(t_0)$--whichever is first--reaches $\emptyset_2$, where t_0 is the time at which observation begins.

One obvious question is, why aren't the effects of $\emptyset_1$ and $\emptyset_3$ simply added together? The answer is that, in our experience with macroobjects or macrophenomena, we find that such causal configurations do not necessarily exhibit additive effects. Consider $\emptyset_1$ to be the water pressure behind a valve, $\emptyset_3$ to be the handle position of a water valve, and $\emptyset_2$ to be the rate at which water is flowing through the valve. If the valve is completely closed and we increase the water pressure by 10 p.s.i., we observe no change in $\emptyset_2$. $\emptyset_2$ is still 0. Therefore, this is an example in the macro-world, of a causal configuration like Figure 9-1 with non-additive effects. Similar relationships could exist in the micro-world of fundamental objects.

After the first impulse reaches $\emptyset_2$ along one of the causal chains, Assumption (8-10) is employed for all other chains which terminate at $\emptyset_2$.

> Assumption (9-1): Between $t_{02}^{\prime}$ and $t_0+\theta_{12;j}$, the values of the pivotal fundamental variables--for chain j of exogenous variable i--are the same on each run, for all i and j.

Assumption (8-10) is also employed for the loops along the various causal chains.

9.3 Causal Chains Converging at $\emptyset_2$ with $\emptyset_2$ Causing Itself Via Causal Loops

Section 9.2 did not consider the possibility that $\emptyset_2$ could cause itself via one or more causal loops. Figure 9-2 represents

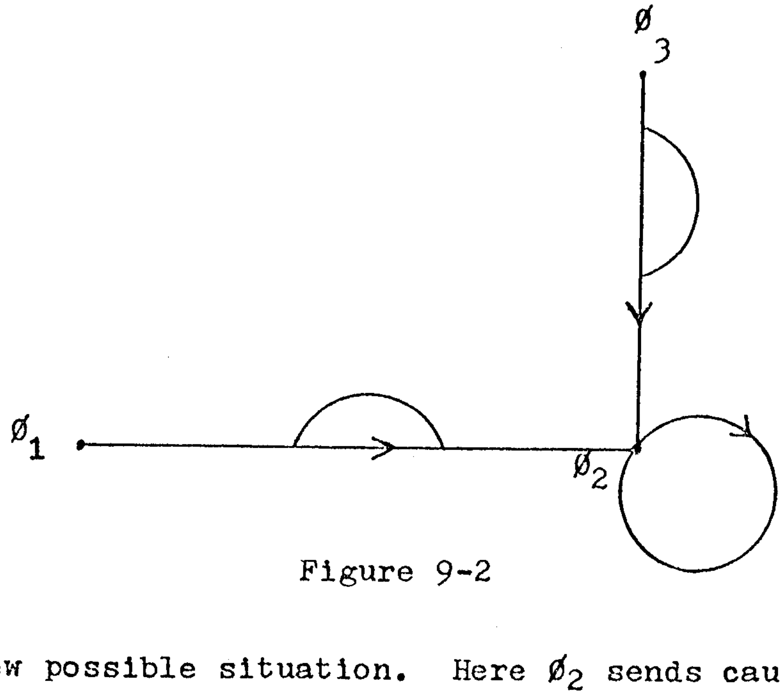

Figure 9-2

this new possible situation. Here $\emptyset_2$ sends causal impulses out along causal loops or chains, which eventually return to $\emptyset_2$, affecting it at some later time.

A discrete equation to describe this phenomenon, i.e., self-causing via looping, can be written as follows:

$$U\left[\emptyset_2(t)\right] \longleftarrow U\left[t-t_{02}',\ \overline{\emptyset_2(t-\theta_{22})},\ \emptyset_2(t_{02}')\right]. \qquad (9\text{-}4)$$

For use in this equation, the definition of t_{02}' must be generalized to also consider impulses from $\emptyset_2$ at t_0. Looking ahead, I should further generalize the definition to consider causal impulses from all of the relevant variables, say m' of them, at t_0. Hence,

> Definition (9-1): t_{02}' is the instant just previous to the time that the first causal impulse--emanating from one of the m' discrete relevant variables at t_0--reaches $\emptyset_2$.

Combining the causes in equation (9-3) with those in (9-4), we get

$$U\left[\emptyset_2(t)\right] \longleftarrow U\left[t-t_{02}',\ \overline{\emptyset_1(t-\theta_{12})},\ \overline{\emptyset_2(t-\theta_{22})},\ \overline{\emptyset_3(t-\theta_{32})},\ \emptyset_2(t_{02}')\right]. \qquad (9\text{-}5)$$

Note that--since U can be altered to generate a prediction of $\overline{\emptyset_2(t-\theta_{22})}$ from previous values of $\emptyset_1$ and $\emptyset_3$ and from $\emptyset_2(t_{02}')$--the inclusion of the $\overline{\emptyset_2(t-\theta_{22})}$ vector, as in equation (9-5), is not essential. But, if the $\overline{\emptyset_2(t-\theta_{22})}$ vector did not appear, the $\overline{\emptyset_1(t-\theta_{12})}$ and $\overline{\emptyset_3(t-\theta_{32})}$ vectors would have to be enlarged to indicated, to the altered U, the time lags with which $\emptyset_1$ and $\emptyset_3$ cause $\emptyset_2$ by way of $\emptyset_2$ to $\emptyset_2$ loops. We will encounter an analogous situation with macrovariables in Chapter 10.

9.4 Converging Causal Chains

Now we discuss the discrete mathematical description of converging causal chains, represented by Figure 9-3. Note the

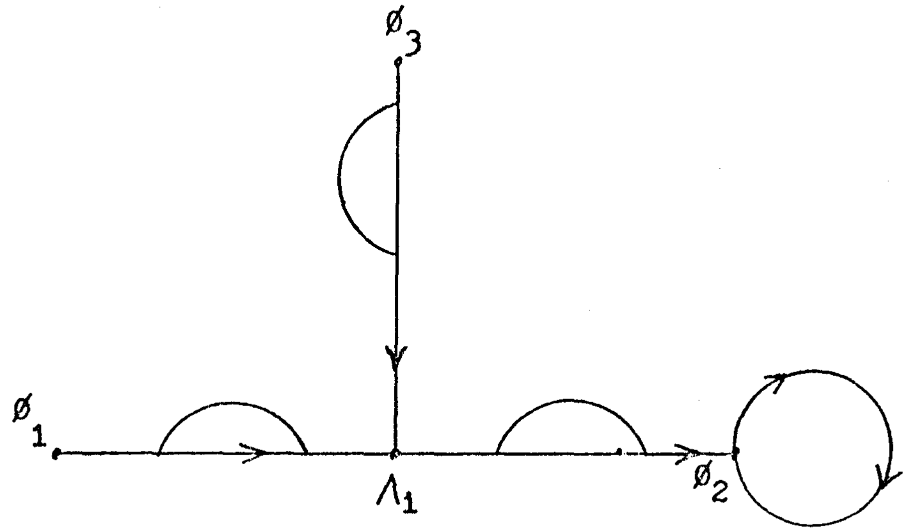

Figure 9-3

similarity between the $\emptyset_1$ to Λ_i and $\emptyset_3$ to Λ_1 part of the figure and Figure 9-1, to which equation (9-3) applies.

By analogy we can write the equation for Λ_i in Figure 9-3 like equation (9-3),

$$U\left[\Lambda_i(t_i)\right] \longleftarrow U\left[t_i - t'_{0i},\ \overline{\emptyset_1(t_i-\theta_{1i})}^{*},\ \overline{\emptyset_3(t_i-\theta_{3j})}^{*},\ \Lambda_i(t'_{0i})\right]. \qquad (9\text{-}6)$$

t_i is just a dummy variable for time and, to arrive at equation (9-8), we will use $t_i = t-\theta_{i2}$. The stars (*'s) in $\overline{\emptyset_1(t_i-\theta_{1i})}^{*}$ and $\overline{\emptyset_3(t_i-\theta_{3i})}^{*}$ are simply intended to distinguish these vectors from $\overline{\emptyset_1(t_i-\theta_{1i})}$ and $\overline{\emptyset_3(t_i-\theta_{3i})}$, which will appear in equation (9-8). Starred and non-starred are simply different vectors.

The causal connection between Λ_1 and ϕ_2 is nothing more than a basic causal chain with a ϕ_2 causal loop. From equation (8-11),

$$U\left[\phi_2(t)\right] \longleftarrow U\left[t-t'_{02},\ \overline{\Lambda_1(t-\theta_{12})},\ \overline{\phi_2(t-\theta_{22})},\ \phi_2(t'_{02})\right]. \qquad (9\text{-}7)$$

Since $\overline{\Lambda_1(t-\theta_{12})}$ is a vector of the values of Λ_1 at various times, we can insert equation (9-6) into (9-7) for each component of the $\overline{\Lambda_1(t-\theta_{12})}$ vector. The resulting equation will contain several different $\overline{\phi_1(t_1-\theta_{11})}^*$ and $\overline{\phi_3(t_1-\theta_{31})}^*$ vectors. But--combining all the ϕ_1 components into one vector($\overline{\phi_1(t-\theta_{12}-\theta_{11})}$ or $\overline{\phi_1(t-\theta_{12})}$)and all the ϕ_3 components into one vector ($\overline{\phi_3(t-\theta_{12}-\theta_{31})}$ or $\overline{\phi_3(t-\theta_{32})}$)--we get

$$U\left[(\phi_2(t)\right] \longleftarrow U\left[t-t'_{02},\ \overline{\phi_1(t-\theta_{12})},\ \overline{\phi_2(t-\theta_{22})},\ \overline{\phi_3(t-\theta_{32})},\ \phi_2(t'_{02})\right]. \qquad (9\text{-}8)$$

The $t_1-t'_{01}$ term from equation (9-6) does not appear in equation (9-8). This is because t'_{02} from equation (9-7) is automatically broadened as a result of Definition (9-1), to include t'_{01}. In equation (9-7), t'_{02} is the instant just prior to the arrival of the first causal impulse from Λ_1. But in equation (9-8), t'_{02} is the instant just prior to the arrival of the first impulse from $\phi_1(t_0)$ or $\phi_3(t_0)$.

The $\Lambda_1(t'_{01})$ term does not appear in equation (9-8), either. When the first causal impulse from either ϕ_1 or ϕ_3 reaches Λ_1, $\Lambda_1(t'_{01}+dt)$ is determined by this causal impulse and the immediately prior value of Λ_i, $\Lambda_i(t'_{0i})$. In our derivation of discrete causal micromathematics, we assumed all immediately prior variables to have the same values on each run and; therefore, incorporated them into the universal operator as empirical constants. The same is done here in arriving at equation (9-8). Although a whole vector of $\Lambda_i(t'_{0i})$ values are inserted into equation (9-7) to arrive at equation (9-8), only the $\Lambda_i(t'_{0i})$ just prior to the arrival of the first causal impulse at Λ_i is assumed the same on each run. Other $\Lambda_i(t'_{0i})$'s are determinable from the initial $\Lambda_i(t'_{0i})$, the subsequent causal impulses, and the pivotal values (as long as the causal impulse along the second chain has not reached Λ_i). Therefore, it need not be assumed that the non-initial $\Lambda_i(t'_{0i})$'s are the same on each run. For the above reasons, no $\Lambda_i(t'_{0i})$'s need be included in equation (9-8).

9.5 Loops Extending from One Simple Causal Chain to Another

We have loops along a causal chain going from one discrete variable to another, but we have not considered loops from one simple causal chain to another (see Figure 9-4). It turns out that

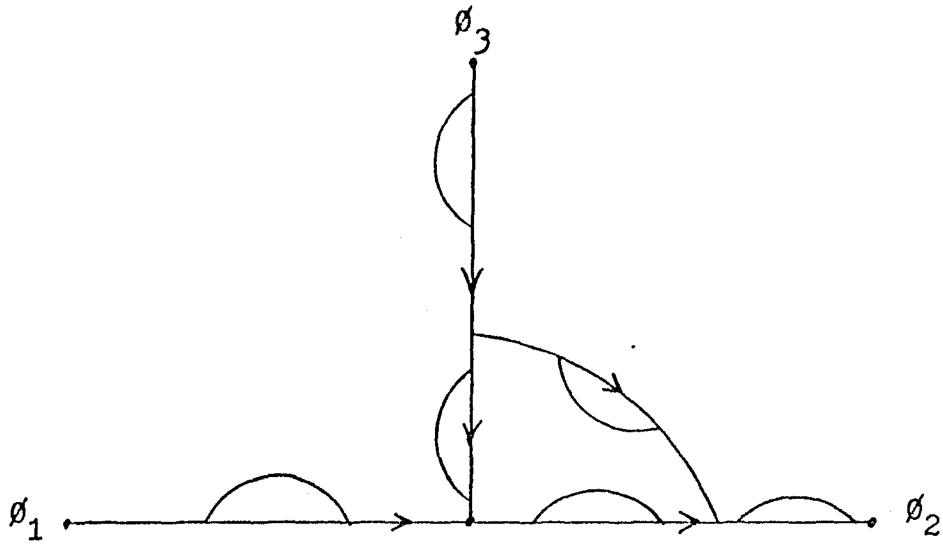

Figure 9-4

the discrete micromathematical description of Figure 9-4 is the same as that of Figure 9-3, viz. equation (9-8). This means that the descriptive equations take on the same operational form, but not that the respective universals operators are the same.

To prove that the discrete micromathematical description of Figure 9-4 is the same as that of Figure 9-3, we break Figure 9-4 into four parts, as in Figure 9-5.

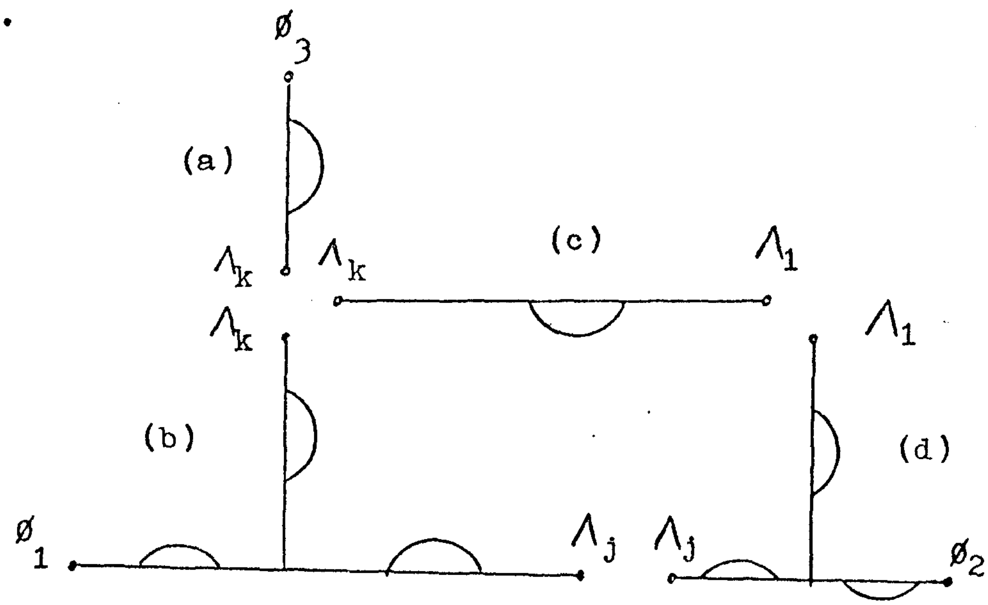

Figure 9-5

As parts (a), (b), (c), and (d) move closer together and make contact with each other, we see that Figure 9-5 is equivalent to Figure 9-4.

Note that the four parts of Figure 9-5 are shapes we have already seen. (a) is the same as the basic causal chain and is, therefore, described by equation (8-11).

$$U\left[\Lambda_k(t_k)\right] \longleftarrow U\left[t_k-t'_{0k},\ \overline{\emptyset_3(t_k-\theta_{3k})}^{*},\ \Lambda_k(t'_{0k})\right]. \tag{9-9}$$

(b) is the same as Figure 9-3 and, therefore, Λ_j is determined according to equation (9-8).

$$U\left[\Lambda_j(t_j)\right] \longleftarrow U\left[t_j-t'_{0j},\ \overline{\emptyset_1(t_j-\theta_{1j})}^{*},\ \overline{\Lambda_k(t_j-\theta_{kj})},\ \Lambda_j(t'_{0j})\right] \tag{9-10}$$

No $\overline{\Lambda_j(t_j-\theta_{jj})}$ term appears in equation (9-10) because this term is optional, as discussed at the end of Section 9.3. It is included in equation (9-8) for greater generality and flexibility, but the effect of $\overline{\Lambda_j(t_j-\theta_{jj})}$ on $\Lambda_j(t_j)$, via a causal loop, can be described as a function of $\emptyset_1$ and Λ_k, which means that $\overline{\Lambda_j(t_j-\theta_{jj})}$ need not appear in equation (9-10). An analogous term was omitted from equation (9-9) and they will continue to be omitted from equations which determine intervening variables.

(c) is of the same form as (a),

$$U\left[\Lambda_1(t_1)\right] \longleftarrow U\left[t_1-t'_{01},\ \overline{\Lambda_k(t_1-\theta_{k1})},\ \Lambda_1(t'_{01})\right], \tag{9-11}$$

and (d) is of the same form as (b),

$$U\left[\phi_2(t_2)\right] \longleftarrow U\left[t_2-t_{02}',\ \overline{\Lambda_j(t_2-\theta_{j2})},\ \overline{\Lambda_1(t_2-\theta_{12})},\ \overline{\phi_2(t_2-\theta_{22})},\ \phi_2(t_{02}')\right]. \qquad (9\text{-}12)$$

Substituting equation (9-9) into (9-10) and (9-11) and then (9-10) and (9-11) into equation (9-12), we get

$$U\left[\phi_2(t_2)\right] \longleftarrow U\left[t_2-t_{02}',\ \overline{\phi_1(t_2-\theta_{12})},\ \overline{\phi_2(t_2-\theta_{22})},\ \overline{\phi_3(t_2-\theta_{32})},\ \phi_2(t_{02}')\right]. \qquad (9\text{-}13)$$

To arrive at (9-13) we made the same notational adjustments as we made in the derivation of equation (9-8).

9.6 Causal Configurations Containing Additional Discrete Exogenous Variables

It is obvious from Figure 9-6 that causal configurations

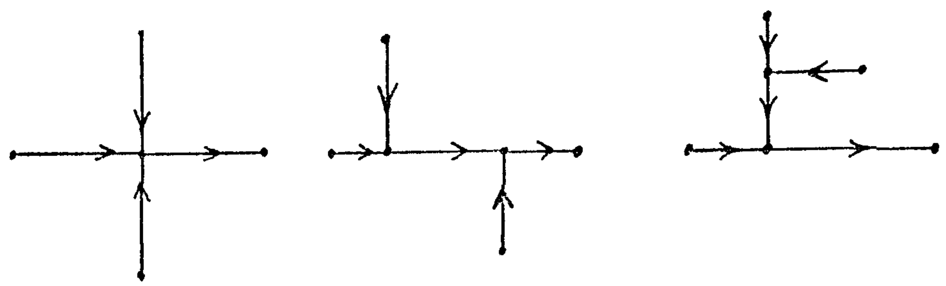

Figure 9-6

containing additional discrete exogenous variables can be broken into pieces which have the shapes shown in Figures 9-1, 9-2, 9-3, or 9-4. Therefore mathematical expressions which describe complex causal configurations can be derived, as was done in Section 9.5.

9.7 The Universal Model of Discrete Causal Micromathematics

The purpose of this section is to write a system of equations which will describe the causal interconnections among m' discrete fundamental variables. This formulation is called the universal model of discrete causal micromathematics.

The extension of the logic of Section 9.5 to m' variables yields

$$U\left[\phi_2(t_2)\right] \longleftarrow U\left[t_2-t'_{02},\ \overline{\phi_1(t_2-\theta_{12})},\ \overline{\phi_2(t_2-\theta_{22})},\ \cdots,\ \overline{\phi_{m'}(t_2-\theta_{m'2})},\ \phi_2(t'_{02})\right] \quad (9\text{-}14)$$

for the causing of ϕ_2. In an analogous manner, equations for the determination of the other m'-1 variables can be written.

Combining these m' equations into a single set, we get the universal model of discrete causal micromathematics.

$$U\left[\phi_1(t_1)\right] \longleftarrow U\left[t_1-t'_{01},\ \overline{\phi_1(t_1-\theta_{11})},\ \overline{\phi_2(t_1-\theta_{21})},\ \cdots,\ \overline{\phi_{m'}(t_1-\theta_{m'1})},\ \phi_1(t'_{01})\right]$$

$$U\left[\phi_2(t_2)\right] \longleftarrow U\left[t_2-t'_{02},\ \overline{\phi_1(t_2-\theta_{12})},\ \overline{\phi_2(t_2-\theta_{22})},\ \cdots,\ \overline{\phi_{m'}(t_2-\theta_{m'2})},\ \phi_2(t'_{02})\right]$$

$$\vdots$$

$$U\left[\phi_{m'}(t_{m'})\right] \longleftarrow U\left[t_{m'}-t'_{0m'},\ \overline{\phi_1(t_{m'}-\theta_{1m'})},\ \cdots,\ \overline{\phi_{m'}(t_{m'}-\theta_{m'm'})},\ \phi_{m'}(t'_{0m'})\right] \quad (9\text{-}15)$$

This model describes the causal interconnections between m' fundamental variables. The next chapter will derive discrete causal macromathematics, which will describe the causal connection between two macro-variables (i.e., two aggregates of fundamental variables).