CHAPTER 11

DERIVATION OF THE UNIVERSAL MODEL OF DISCRETE CAUSAL MACROMATHEMATICS

Sections

11.1 Introduction

The purpose of Chapter 11 is analogous to the purpose of Chapter 9--where a mathematical formulation involving one exogenous and one endogenous variable was generalized to form an m' variable, reciprocal causal model. The difference is that the present derivation involves a formulation representing causal connections among macrovariables via causal macrochains rather than causal connections among fundamental variables via causal chains.

Even though Chapters 9 and 11 are similar in purpose, Chapter 11 is analogous to Chapter 10 in the derivation technique employed. The derivation of this chapter repeats the derivation process of Chapter 10, except here we assume n exogenous variables rather than one. Employing, conceptually, the derivation process of Chapter 10 to determine the discrete causal equation for each macrovariable in the system, we obtain the universal model of discrete causal macromathematics.

11.2 Derivation

Consider n groups of fundamental variables, shown in Figure 11-1. The lines in the figure represent causal chains of which an infinite number are possible.

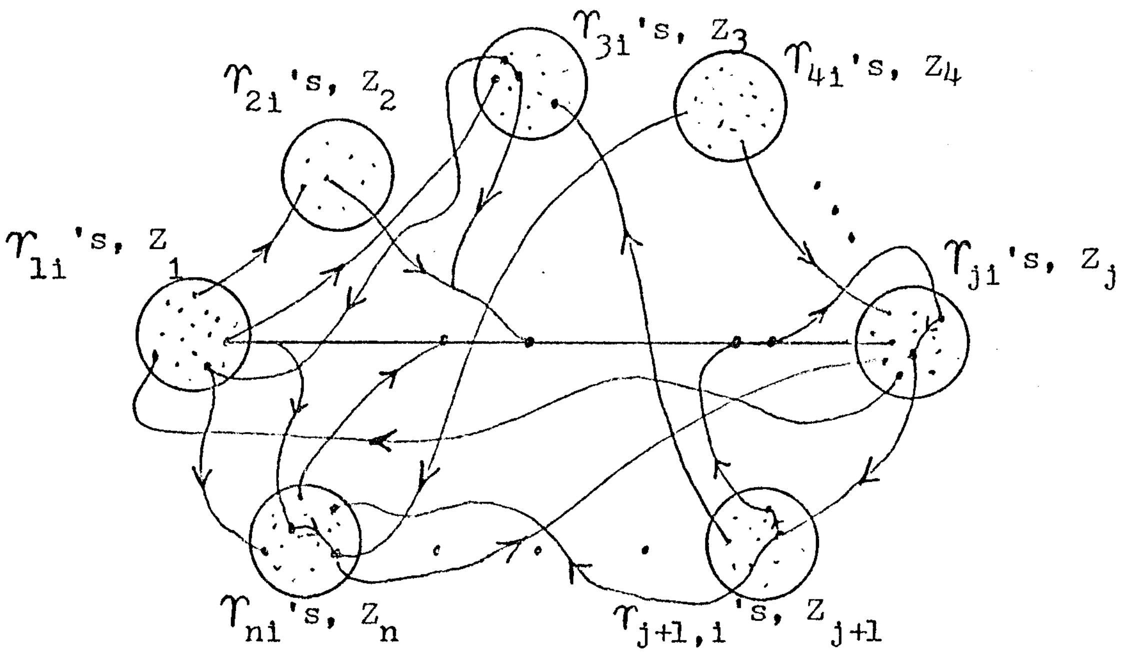

Figure 11-1

To describe the effect of all the fundamental variables in the system on the fundamental variables composing Z_j, we can make use of equation (9-15.i) to get

$$U\left[\Upsilon_{ji}(t)\right] \leftarrow U\left[t-t'_{0i},\ \overline{\Upsilon_{11}(t-\theta_{11,ji})},\ \overline{\Upsilon_{12}(t-\theta_{12,ji})},\ \ldots,\ \overline{\Upsilon_{1p_1}(t-\theta_{1p_1,ji})},\ \overline{\Upsilon_{21}(t-\theta_{21,ji})},\ \overline{\Upsilon_{22}(t-\theta_{22,ji})},\ \ldots,\ \overline{\Upsilon_{2p_2}(t-\theta_{2p_2,ji})},\ \ldots,\ \overline{\Upsilon_{n1}(t-\theta_{n1,ji})},\ \overline{\Upsilon_{n2}(t-\theta_{n2,ji})},\ \ldots,\ \overline{\Upsilon_{np_n}(t-\theta_{np_n,ji})},\right.$$

$$\Upsilon_{ji}(t'_{0i}) \text{ , for } i = 1,2, \ldots, p_j. \qquad (11\text{-}1)$$

t'_{0i} is the time, an instant (dt) before the first causal impulse--produced at t_0 by one of the outside variables--reaches Υ_{ji}. p_j is the number of fundamental variables composing Z_j. Equations (11-1) assume

> Assumption (11-1): that all relevant variables are considered. In other words we assume that all relevant variables are shown in Figure 11-1.

Now, we aggregate the component fundamental variables of Z_j via some mathematical operation, as was done in Chapter 10, to get $U\left[\Upsilon_{j1}(t), \Upsilon_{j2}(t), \ldots, \Upsilon_{jp_j}(t)\right]$. Combining the causing sides of equations (11-1) via this aggregation operator and equating the two aggregates, we get

$$U\left[\Upsilon_{j1}(t), \Upsilon_{j2}(t), \ldots, \Upsilon_{jp_j}(t)\right] \longleftarrow U\left[t-t_{0j}, \overline{\Upsilon_{11}(t-\theta''''_{11,j})}, \overline{\Upsilon_{12}(t-\theta''''_{12,j})}, \ldots, \overline{\Upsilon_{1p_1}(t-\theta''''_{1p_1,j})}, \overline{\Upsilon_{21}(t-\theta''''_{21,j})}, \ldots, \overline{\Upsilon_{np_n}(t-\theta''''_{np_n,j})}, \Upsilon_{j1}(t_{0j}), \Upsilon_{j2}(t_{0j}), \ldots, \Upsilon_{jp_j}(t_{0j})\right], \qquad (11\text{-}2)$$

where $\overline{\theta''''_{ki,j}}$ is a vector of all direct causal time lags running from Υ_{ki} to any component of Z_j. t_{0j} is the instant (dt) before the first causal impulse reaches Z_j. t_{0j} is the smallest t'_{0i}.

Note that the t'_{0i}'s, for $i = 1,2, \ldots, p_j$, which appear in equations (11-1), have been replaced in equation (11-2) by t_{0j}'s. This is because, in equation (11-2), we are interested in the instant before the first impulse reaches Z_j rather than the instant before the first impulses reach each component of Z_j. This interest is due to the fact that, in equation (11-2), we are moving toward dealing with macroobjects rather than fundamental objects (see Subsection 10.4.1).

Now, as in Chapter 10, we must define the macrovariables (Z's), apply Assumption (10-5), make Assumptions (10-4) and/or (10-3), and replace the Υ's by Z's. Defining the Z's, we get

$$\begin{aligned} U[Z_1(t)] &\equiv U[\Upsilon_{11}(t), \Upsilon_{12}(t), \ldots, \Upsilon_{1p_1}(t)] \\ U[Z_2(t)] &\equiv U[\Upsilon_{21}(t), \Upsilon_{22}(t), \ldots, \Upsilon_{2p_2}(t)] \\ &\vdots \\ U[Z_n(t)] &\equiv U[\Upsilon_{n1}(t), \Upsilon_{n2}(t), \ldots, \Upsilon_{np_n}(t)]. \end{aligned} \qquad (11\text{-}3)$$

If we make Assumption (10-4) or (10-3) about $\Upsilon_{ji}(t)$, for all i, on the caused side of equation (11-2), they can be replaced by $Z_j(t)$. Making Assumption (10-5) about the time lags on the causing side of equation (11-2), enables us to aggregate the causal chains into a number of causal macrochains. If we then employ Assumption (10-4) and/or (10-3), as appropriate, to the causing vectors in equation (11-2), the vectors can be replaced by $\overline{Z_1(t-\tau_{12})}$, $\overline{Z_2(t-\tau_{22})}$, ..., $\overline{Z_n(t-\tau_{n2})}$. Remember that $\overline{\tau_{kl}}$ is the vector of time lags for causal macrochains from Z_k to Z_l. If we make Assumption (10-4) or (10-3) concerning the last p_j terms on the causing side of equation (11-2), they can be replaced by $Z_j(t_{0j})$. Applying all these assumptions and substitutions to equation (11-2), we get

$$\begin{aligned} U[Z_j(t)] \longleftarrow U\big[&t-t_{0j}, \overline{Z_1(t-\tau_{1j})}, \overline{Z_2(t-\tau_{2j})}, \\ &\ldots, \overline{Z_n(t-\tau_{nj})}, Z_j(t_{0j})\big]. \end{aligned} \qquad (11\text{-}4)$$

Equation (11-4) describes the determination of $Z_j(t)$. It is obvious that the causal equation for any Z can be derived via this procedure since the derivation is valid for $j = 1,2, \ldots, n$. Therefore, we obtain a system of equations which determine all Z's.

$$
\begin{aligned}
U[Z_1(t)] &\longleftarrow U[t-t_{01}, \overline{Z_1(t-\tau_{11})}, \overline{Z_2(t-\tau_{21})}, \\
&\quad \ldots, \overline{Z_n(t-\tau_{n1})}, Z_1(t_{01})] \\
U[Z_2(t)] &\longleftarrow U[t-t_{02}, \overline{Z_1(t-\tau_{12})}, \overline{Z_2(t-\tau_{22})}, \\
&\quad \ldots, \overline{Z_n(t-\tau_{n2})}, Z_2(t_{02})] \\
&\vdots \\
U[Z_n(t)] &\longleftarrow U[t-t_{0n}, \overline{Z_1(t-\tau_{1n})}, \overline{Z_2(t-\tau_{2n})}, \\
&\quad \ldots, \overline{Z_n(t-\tau_{nn})}, Z_n(t_{0n})]. \qquad (11\text{-}5)
\end{aligned}
$$

This is the universal model of discrete causal macro-mathematics.