

FOUNDATIONS OF MATHEMATICAL EPISTEMOLOGY:
A Derivation of Causal Statistics

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Ph.D. Dissertation

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Of course, the mistakes in this work are my own, and I hope that some will eventually be interesting enough to deserve refutation.

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ABSTRACT

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Causal statistics is a mathematical inquiring system which enables empirical researchers to draw causal inferences from non-experimental, quasi-experimental, and imperfectly experimental data; based upon the least restrictive assumptions possible. The non-experimental sciences (e.g., the social sciences, some medical sciences, etc.) desperately need such a tool. The development of causal statistics may be as important to the non-experimental sciences as the codification of the scientific method was to the physical (i.e., experimental) sciences.

This dissertation is the initial phase of a larger research effort, called the Causal Statistics Project. The objective of the Causal Statistics Project is the derivation, operationalization, computerization, and application of causal statistics. The dissertation analyzes the philosophical background of the concept of causality and derives causal statistics.

Long standing philosophical definitions of causality are presented, analyzed, and rejected. To fill the void a new, theoretical definition of cause--which is applicable to causal statistics--is constructed. This is followed by an analysis of the causal philosophies of the two most influential causal philosophers, David Hume and John Stuart Mill.

The derivation of causal statistics begins by presenting a causal explanation of the operation of the universe. This cosmology is axiomatized, primitives are noted, and an object language is erected. From these foundations a lengthy mathematical derivation is performed, eventually resulting in the universal model of causal statistics.

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